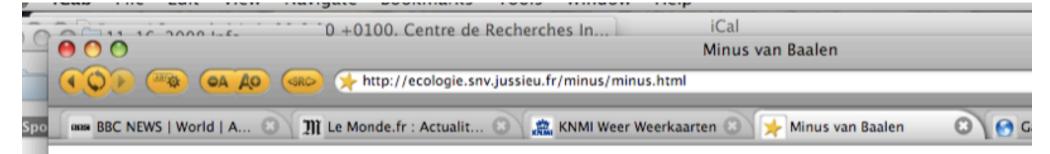
FdV Jan 2013 The Interaction Between **Evolution and Ecology**

Minus van Baalen (CNRS, UMR 7625 EcoEvo, Paris)

Who am I

Minus van Baalen

- Researcher at the CNRS
- Ex-head of UMR 7625 « Ecologie et Evolution »
- Dutch
- Thesis Evolutionary Biology 1994
- Theoretician



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Research Interests

Ecology and evolution

When mutant individuals with a changed trait are successful, they will increase in numbers and thus start affecting population dynamics, resource availability, the prevalence of parasites, the intensity of interspecific competition, community structure, and so on. Together, these effects will cause a feedback because ecological parameters will, in general, affect the traits that are favoured. More on this can be found in the introduction to my thesis.

Interacting populations

Such eco-evolutionary feedback loops will be particularly intense in systems with interacting populations: adaptation and counteradaptation often have population dynamical consequences. A good example is the evolution of virulence. If avirulent parasites are common, host density increases and, with it, the force of infection. But so does the intensity of within-host competition, which favours more virulent parasites. Ecological effects will modify or sometimes even revert selection pressure on virulence.

Space

How spatial dynamics affect evolution (and vice versa) is still poorly understood. When a mutant invades a 'viscous' system it typically does so in the form of an expanding cluster of relatives. Ultimately it is therefore the characteristics of these clusters that determine whether the invasion will be successful. On other words, the unit of selection in a viscous systems is a cluster of

Research Interests

- Ecology and evolution
- Interacting populations
- Space
- Dangerous Liaisons
- Communication
- Kin selection, coloniality and disease
- Immune functioning and virulence

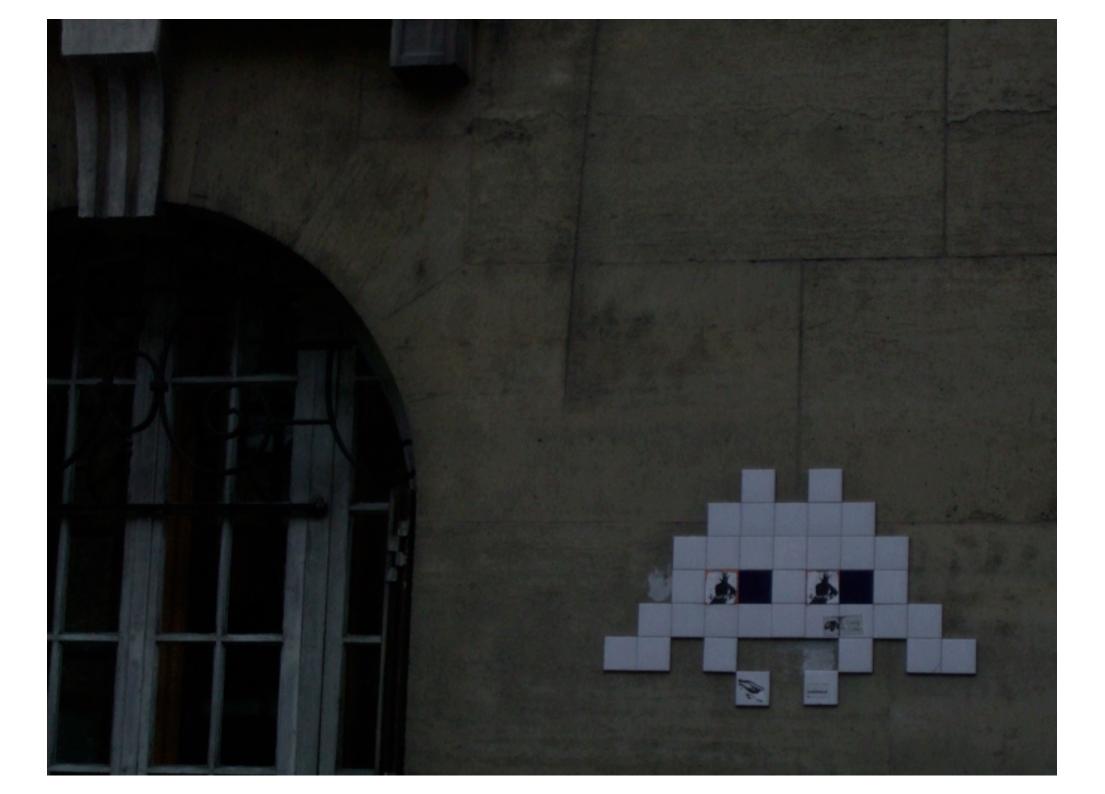
Ecology+Evolution=

- Population Genetics
- Game Theory
- Life History Theory
- Community Ecology



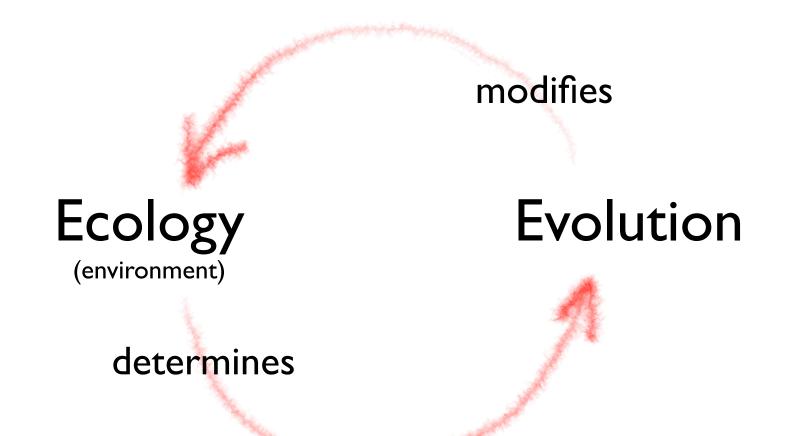
Adaptive Dynamics

Invasion Invasion as a unifying conceptual tool





Eco-Evolutionary Feedback



Evolution

History

Before 1800

- various theories of evolution
- species evolve

Lamarck, Erasmus Darwin

After 1800

mechanism: natural selection

Charles Darwin, Alfred R. Wallace

THE ORIGIN OF SPECIES

BY MEANS OF NATURAL SELECTION,

OR THE

PRESERVATION OF FAVOURED RACES IN THE STRUGGLE FOR LIFE.

By CHARLES DARWIN, M.A.,

FELLOW OF THE ROYAL, GEOLOGICAL, LINNÆAN, ETC., SOCIETIES;
AUTHOR OF 'JOURNAL OF RESEARCHES DURING H. M. S. BEAGLE'S VOYAGE
ROUND THE WORLD.'

LONDON:

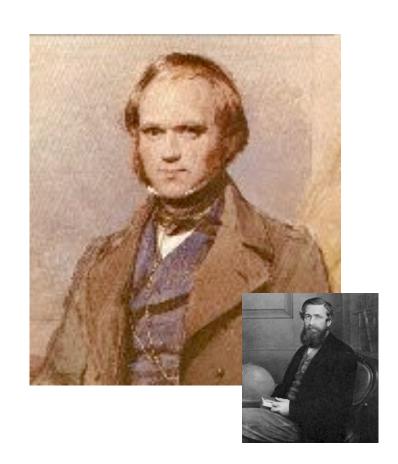
JOHN MURRAY, ALBEMARLE STREET.

1859.

Darwin's Insight

(& Wallace's)

- + Reproduction generates variation
- + Individuals compete
- + Traits affect individuals' differential survival
- = 'Evolution by Natural Selection'



Rediscovery of Mendel

Early 1900s

- rediscovery of Mendel's work
- phenotypes change because genotypes change
- genes remain the same
 - no evolutionary change

Synthesis

Genes are not fixed

rare mutations modify genes

Hugo de Vries

'Neo-Darwinian Synthesis'

fixation of mutations

Ronald A. Fischer

Invasion as a unifying conceptual tool in ecology and evolution

Minus van Baalen (CNRS, UMR 7625 EcoEvo, Paris)

Invasion

Invasion is a notion that underpins

- Population Genetics
- Game Theory
- Life History Theory
- Community Ecology

Invasion

Notions of invasion underpin

- Population Genetics
- Game Theory
- Life History Theory
- Community Ecology

All organisms grow, reproduce and eventually die What is the result:

- a growing population?
- extinction?

Need to integrate life-history components

Hal Caswell

Evolutionary Life History Theory

All organisms grow, reproduce and eventually die

Given finite resources, how should an individual invest in growth, reproduction and survival

Kooijman

Since 1960s: Evolutionary Life History Theory

Eric Charnov, Steve Stearns

Population-level view:

- Net rate of reproduction: r = b d
 - where the rates of reproduction b and mortality d may depend on environmental conditions
- \blacksquare A population invades if (and only if) r is positive

Individual-level view

- A population increases on average an individual has more than one offspring
- Average lifetime: 1/d
- **Expected lifetime reproductive success** or 'Basic Reproduction Ratio' $R_0 = b/d$
- **Invasion** if (and only if) $R_0 > 1$

Hypothesis

- Natural Selection maximizes $R_0 = b/d$
- Basic Reproduction Ratio

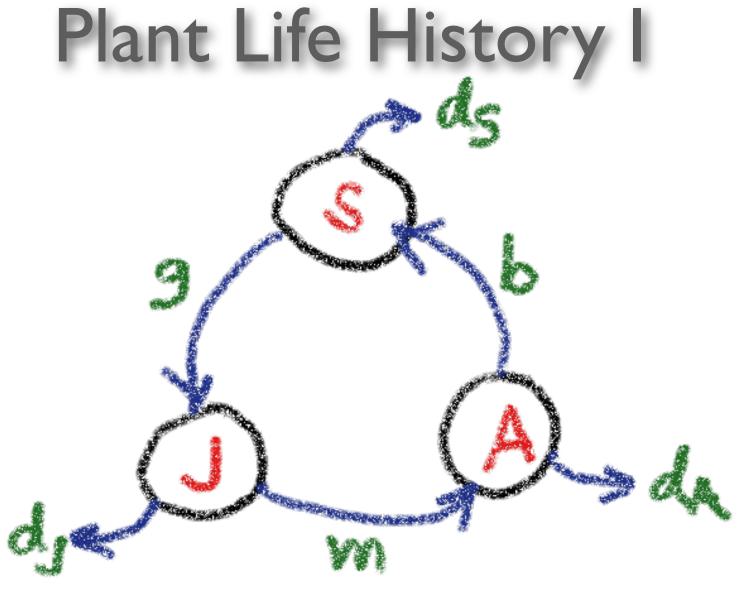
Most theory is about how individuals might achieve this

Caricature

Individuals try to maximize their lifetime reproductive success by adopting the optimal allocation of resources into reproduction and survival.

Plant Life History I

- Continuous time
- Three stages
 - Seeds S
 - Juveniles (non-reproducing) J
 - Adults (reproducing) A



Plant Life History I

$$\frac{dS}{dt} = bA - d_3S - gS$$

$$\frac{dJ}{dt} = gS - d_3J - m_J$$

$$\frac{dA}{dt} = m_J - d_AA$$

Plant Life History I

$$\frac{d}{dt}\begin{pmatrix} S \\ J \end{pmatrix} = \begin{pmatrix} -d_{3}-g & 0 & b \\ g & -d_{3}-m & 0 \\ 0 & m & -d_{4}\end{pmatrix}\begin{pmatrix} S \\ J \\ A \end{pmatrix}$$

Linear model

Solution
$$\chi(\xi) = \sum_{i=1}^{n} c_i U_i e^{\lambda_i \xi}$$

 U_i i-th eigenvector λ_i i-th eigenvalue

Dominant eigenvalue λ

Solution converges to
$$\chi(t) \propto Ue^{\lambda t}$$

Population increases if $\lambda > 0$, decreases if $\lambda < 0$

```
Out[3]= \left\{\left\{\lambda \to \frac{\pi}{c}\left(-2\left(g+m+d_{H}+d_{J}+d_{J}\right)-\left(22^{1/3}\left(g^{2}-gm+m^{2}+d_{H}^{2}+d_{J}^{2}+2gd_{J}-md_{J}+d_{J}^{2}-d_{J}\left(g-2m+d_{J}\right)-d_{H}\left(g+m+d_{J}+d_{J}\right)\right)\right\}\right\}
                                                                                                                       3\,d_{0}^{2}\,d_{J}-3\,g\,d_{T}^{2}+6\,n\,d_{T}^{2}-3\,d_{0}\,d_{T}^{2}+2\,d_{T}^{3}+6\,g^{2}\,d_{S}-6\,g\,n\,d_{S}-3\,n^{2}\,d_{S}-6\,g\,d_{0}\,d_{S}+12\,n\,d_{0}\,d_{S}-3\,d_{0}^{2}\,d_{S}-6\,g\,d_{T}\,d_{S}-6\,n\,d_{T}\,d_{S}+12\,d_{0}\,d_{T}\,d_{S}+12\,d_{0}\,d_{T}\,d_{S}+12\,d_{0}\,d_{T}\,d_{S}+12\,d_{0}\,d_{T}\,d_{S}+12\,d_{0}\,d_{T}\,d_{S}+12\,d_{0}\,d_{T}\,d_{S}+12\,d_{0}\,d_{T}\,d_{S}+12\,d_{0}\,d_{T}\,d_{S}+12\,d_{0}\,d_{T}\,d_{S}+12\,d_{0}\,d_{T}\,d_{S}+12\,d_{0}\,d_{T}\,d_{S}+12\,d_{0}\,d_{T}\,d_{S}+12\,d_{0}\,d_{T}\,d_{S}+12\,d_{0}\,d_{T}\,d_{S}+12\,d_{0}\,d_{T}\,d_{S}+12\,d_{0}\,d_{T}\,d_{S}+12\,d_{0}\,d_{T}\,d_{S}+12\,d_{0}\,d_{T}\,d_{S}+12\,d_{0}\,d_{T}\,d_{S}+12\,d_{0}\,d_{T}\,d_{S}+12\,d_{0}\,d_{T}\,d_{S}+12\,d_{0}\,d_{T}\,d_{S}+12\,d_{0}\,d_{T}\,d_{S}+12\,d_{0}\,d_{T}\,d_{S}+12\,d_{0}\,d_{T}\,d_{S}+12\,d_{0}\,d_{T}\,d_{S}+12\,d_{0}\,d_{T}\,d_{S}+12\,d_{0}\,d_{T}\,d_{S}+12\,d_{0}\,d_{T}\,d_{S}+12\,d_{0}\,d_{T}\,d_{S}+12\,d_{0}\,d_{T}\,d_{S}+12\,d_{0}\,d_{T}\,d_{S}+12\,d_{0}\,d_{T}\,d_{S}+12\,d_{0}\,d_{T}\,d_{S}+12\,d_{0}\,d_{T}\,d_{S}+12\,d_{0}\,d_{T}\,d_{S}+12\,d_{0}\,d_{T}\,d_{S}+12\,d_{0}\,d_{T}\,d_{S}+12\,d_{0}\,d_{T}\,d_{S}+12\,d_{0}\,d_{T}\,d_{S}+12\,d_{0}\,d_{T}\,d_{S}+12\,d_{0}\,d_{T}\,d_{S}+12\,d_{0}\,d_{T}\,d_{S}+12\,d_{0}\,d_{T}\,d_{S}+12\,d_{0}\,d_{T}\,d_{S}+12\,d_{0}\,d_{T}\,d_{S}+12\,d_{0}\,d_{T}\,d_{S}+12\,d_{0}\,d_{T}\,d_{S}+12\,d_{0}\,d_{T}\,d_{S}+12\,d_{0}\,d_{T}\,d_{S}+12\,d_{0}\,d_{T}\,d_{S}+12\,d_{0}\,d_{T}\,d_{S}+12\,d_{0}\,d_{T}\,d_{S}+12\,d_{0}\,d_{T}\,d_{S}+12\,d_{0}\,d_{T}\,d_{S}+12\,d_{0}\,d_{T}\,d_{S}+12\,d_{0}\,d_{T}\,d_{S}+12\,d_{0}\,d_{T}\,d_{S}+12\,d_{0}\,d_{T}\,d_{S}+12\,d_{0}\,d_{T}\,d_{S}+12\,d_{0}\,d_{T}\,d_{S}+12\,d_{0}\,d_{T}\,d_{S}+12\,d_{0}\,d_{T}\,d_{S}+12\,d_{0}\,d_{T}\,d_{S}+12\,d_{0}\,d_{T}\,d_{S}+12\,d_{0}\,d_{T}\,d_{S}+12\,d_{0}\,d_{T}\,d_{S}+12\,d_{0}\,d_{T}\,d_{S}+12\,d_{0}\,d_{T}\,d_{S}+12\,d_{0}\,d_{T}\,d_{S}+12\,d_{0}\,d_{T}\,d_{S}+12\,d_{0}\,d_{T}\,d_{S}+12\,d_{0}\,d_{T}\,d_{S}+12\,d_{0}\,d_{T}\,d_{S}+12\,d_{0}\,d_{T}\,d_{S}+12\,d_{0}\,d_{T}\,d_{S}+12\,d_{0}\,d_{T}\,d_{S}+12\,d_{0}\,d_{T}\,d_{S}+12\,d_{0}\,d_{T}\,d_{S}+12\,d_{0}\,d_{T}\,d_{S}+12\,d_{0}\,d_{T}\,d_{S}+12\,d_{0}\,d_{T}\,d_{S}+12\,d_{0}\,d_{T}\,d_{S}+12\,d_{0}\,d_{T}\,d_{S}+12\,d_{0}\,d_{T}\,d_{S}+12\,d_{0}\,d_{T}\,d_{S}+12\,d_{0}\,d_{T}\,d_{S}+12\,d_{0}\,d_{T}\,d_{
                                                                                                                                                3d_1^2d_3 + 6gd_3^2 - 3nd_3^2 - 3d_0d_3^2 - 3d_1d_3^2 + 2d_3^2 + \sqrt{(-4(g^2 - gn + n^2 + d_0^2 + d_0^2 + 2gd_3 - nd_0 + d_0^2 - d_1(g - 2n + d_0))^3 + (g + n + d_0 + d_0^2 +
                                                                                                                                                                               (-2g^3 + 27bgn + 3g^2n + 3gn^2 - 2n^3 - 2d_0^3 - 2d_0^3 - 6g^2d_3 + 6gnd_3 + 3n^2d_3 - 6gd_3^2 + 3nd_3^2 - 2d_3^3 + 3d_0^2(g - 2n + d_3) + 3d_0^2(g + n + d_0 + d_0) + 3d_0^2(g + n + d_0) + 3d_0^
                                                                                                                                                                                                         3d_{B}(g^{2}-4gn+n^{2}+d_{x}^{2}+d_{y}(-4g+2n-4d_{y})+2(g-2n)d_{y}+d_{y}^{2})+3d_{y}(g^{2}+2gn-2n^{2}+2(g+n)d_{y}+d_{y}^{2})^{2})^{1/3}
                                                                                                            3d_1^2d_3 + 6gd_3^2 - 3nd_3^2 - 3d_0d_3^2 - 3d_0d_3^2 + 2d_3^2 + \sqrt{(-4(g^2 - gn + n^2 + d_0^2 + 2gd_3 - md_3 + d_3^2 - d_1(g - 2n + d_3) - d_0(g + m + d_1 + d_3))^3} + \frac{1}{2}
                                                                                                                                                                              (-2g^3 + 27bgn + 3g^2n + 3gn^2 - 2n^3 - 2d_0^3 - 2d_0^3 - 2d_0^3 - 6g^2d_0 + 6gnd_0 + 3n^2d_0 - 6gd_0^2 + 3nd_0^2 - 2d_0^3 + 3d_0^2(g - 2n + d_0) + 3d_0^2(g + n + d_0 + d_0)
                                                                                                                                                                                                          3d_{B}\left(g^{2}-4gn+n^{2}+d_{x}^{2}+d_{y}\left(-4g+2n-4d_{y}\right)+2\left(g-2n\right)d_{y}+d_{y}^{2}\right)+3d_{y}\left(g^{2}+2gn-2n^{2}+2\left(g+n\right)d_{y}+d_{y}^{2}\right)^{2}\right)^{1/3}\right)\right\}.
                                                             \left\{\lambda \to \frac{1}{42} \left(-4 \left(g + m + d_{H} + d_{J} + d_{S}\right) + \left(2 2^{1/3} \left(1 + i \sqrt{3}\right) \left(g^{2} - g m + m^{2} + d_{H}^{2} + d_{J}^{2} + 2 g d_{S} - m d_{S} + d_{S}^{2} - d_{J} \left(g - 2 m + d_{S}\right) - d_{H} \left(g + m + d_{J} + d_{J}\right)\right)\right\} / d_{H} + d_{H}
                                                                                                                       3\,d_{0}^{2}\,d_{J} - 3\,g\,d_{J}^{2} + 6\,m\,d_{J}^{2} - 3\,d_{0}\,d_{J}^{2} + 2\,d_{J}^{3} + 6\,g^{2}\,d_{3} - 6\,g\,m\,d_{3} - 3\,n^{2}\,d_{3} - 6\,g\,d_{0}\,d_{3} + 12\,m\,d_{0}\,d_{3} - 3\,d_{0}^{2}\,d_{3} - 6\,g\,d_{J}\,d_{3} - 6\,m\,d_{J}\,d_{3} + 12\,d_{0}\,d_{J}\,d_{3} - 4\,g\,d_{0}\,d_{3} + 12\,d_{0}\,d_{J}\,d_{3} + 12\,d_{0}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,d_{J}\,
                                                                                                                                                  3d_1^2d_3 + 6gd_3^2 - 3nd_3^2 - 3d_4d_3^2 - 3d_4d_3^2 + 2d_3^2 + \sqrt{(-4(g^2 - gn + n^2 + d_4^2 + d_3^2 + 2gd_5 - md_5 + d_3^2 - d_3(g - 2n + d_5) - d_6(g + m + d_3 + d_3))^3} + \frac{3d_1^2d_3 + 6gd_3^2 - 3nd_3^2 - 3d_4d_3^2 - 3d_3^2 + 2d_3^2 + \sqrt{(-4(g^2 - gn + n^2 + d_4^2 + d_3^2 + 2gd_5 - md_5 + d_3^2 - d_3(g - 2n + d_5)) - d_6(g + m + d_3 + d_3))^3}
                                                                                                                                                                              (-2g^3 + 27bgn + 3g^2n + 3gn^2 - 2n^3 - 2d_0^3 - 2d_0^3 - 6g^2d_3 + 6gnd_3 + 3n^2d_3 - 6gd_3^2 + 3nd_3^2 - 2d_3^3 + 3d_1^2(g - 2n + d_3) + 3d_0^2(g + n + d_1 + d_2 + d_3) + 3d_0^2(g + n + d_2 + d_3) + 3d_0^2(g + n + d_3 + d_3) + 3d_0^2(g + n + d_3) + 3d_0^2(g + 
                                                                                                                                                                                                          3d_{\theta}(q^2-4qn+n^2+d_T^2+d_T(-4q+2n-4d_S)+2(q-2m)d_S+d_S^2)+3d_T(q^2+2qn-2m^2+2(q+m)d_S+d_S^2)^2)^{1/3}+
                                                                                                            2^{2/3} (1 - i \sqrt{3}) (2g^3 - 27bgn - 3g^2n - 3gn^2 + 2n^3 - 3g^2d_0 + 12gnd_0 - 3n^2d_0 - 3gd_0^2 - 3nd_0^2 + 2d_0^3 - 3g^2d_0 - 6gnd_0 + 6n^2d_0 + 12gd_0d_0 - 3g^2d_0 + 2d_0^3 - 3g^2d_0 - 3gd_0^2 + 2d_0^3 - 3g^2d_0 - 3g
                                                                                                                                                  6 \text{ md}_{\theta} d_{x} - 3 d_{\theta}^{2} d_{x} - 3 q d_{x}^{2} + 6 \text{ md}_{x}^{2} - 3 d_{\theta} d_{x}^{2} + 2 d_{x}^{3} + 6 q^{2} d_{x} - 6 q m d_{x} - 3 m^{2} d_{x} - 6 q d_{\theta} d_{x} + 12 m d_{\theta} d_{x} - 3 d_{\theta}^{2} d_{x} - 6 q d_{x} d_{x} - 6 m d_{x} d_{x} + 12 d_{\theta} d_{x} d_{x} - 6 q d_{x
                                                                                                                                                  3d_{3}^{2}d_{5} + 6gd_{3}^{2} - 3md_{3}^{2} - 3d_{6}d_{3}^{2} - 3d_{7}d_{3}^{2} + 2d_{3}^{2} + \sqrt{\left(-4\left(g^{2} - gn + n^{2} + d_{6}^{2} + d_{3}^{2} + 2gd_{5} - md_{5} + d_{3}^{2} - d_{7}\left(g - 2n + d_{5}\right) - d_{6}\left(g + n + d_{7} + d_{7}\right)\right)^{3}} + \frac{3d_{7}^{2}d_{5}}{2} + \frac{3d_{7}^{2}d_{5}^{2} - 3md_{7}^{2} - 3d_{7}d_{7}^{2} - 3d_{7}d_{7}^{2} + 2d_{7}^{2} - 3d_{7}d_{7}^{2}}{2} + \frac{3d_{7}^{2}d_{7}^{2} - 3d_{7}d_{7}^{2} - 3d_{7}d_{7}^{2}}{2} + \frac{3d_{7}^{2}d_{7}^{2} - 3d_{7}d_{7}^{2} - 3d_{7}d_{7}^{2}}{2} + \frac{3d_{7}^{2}d_{7}^{2} - 3d_{7}d_{7}^{2} - 3d_{7}d_{7}^{2}}{2} + \frac{3d_{7}^{2}d_{7}^{2} - 3d_{7}d_{7}^{2}}{2} + \frac{3d_{7}^{2}d_{7}^{2} - 3d_{7}d_{7}^{2}}{2} + \frac{3d_{7}^{2}d_{7}^{2} - 3d_{7}^{2}d_{7}^{2}}{2} + \frac{3d_{7}^{2}d_{7}^{2} - 3d_{7}^{2}d_{7}^{2}}{2} + \frac{3d_{7}^{2}d_{7}^{2} - 3d_{7}^{2}d_{7}^{2}}{2} + \frac{3d_{7}^{2}d_{7}^{2}}{2} + 
                                                                                                                                                                              (-2g^3 + 27bgn + 3g^2n + 3gn^2 - 2n^3 - 2d_0^3 - 2d_0^3 - 6g^2d_3 + 6gnd_3 + 3n^2d_3 - 6gd_3^2 + 3nd_3^2 - 2d_3^3 + 3d_4^2(g - 2n + d_3) + 3d_4^2(g + n + d_4 + 
                                                                                                                                                                                                          3d_{R}(g^{2}-4gn+n^{2}+d_{J}^{2}+d_{J}(-4g+2n-4d_{S})+2(g-2n)d_{S}+d_{S}^{2})+3d_{J}(g^{2}+2gn-2n^{2}+2(g+n)d_{S}+d_{S}^{2}))^{2})^{1/3})
                                                             \left\{\lambda \to \frac{1}{42} \left(-4 \left(g + m + d_{A} + d_{J} + d_{3}\right) + \left(2 2^{1/3} \left(1 - i \sqrt{3}\right) \left(g^{2} - g m + m^{2} + d_{A}^{2} + d_{J}^{2} + 2 g d_{3} - m d_{3} + d_{3}^{2} - d_{J} \left(g - 2 m + d_{3}\right) - d_{A} \left(g + m + d_{J} + d_{J}\right)\right)\right) \right/ d_{A} + d_{A
                                                                                                                     3\,d_{0}^{2}\,d_{3} - 3\,g\,d_{3}^{2} + 6\,m\,d_{3}^{2} - 3\,d_{0}\,d_{3}^{2} + 2\,d_{3}^{3} + 6\,g^{2}\,d_{3} - 6\,g\,m\,d_{3} - 3\,n^{2}\,d_{3} - 6\,g\,d_{0}\,d_{3} + 12\,n\,d_{0}\,d_{3} - 3\,d_{0}^{2}\,d_{3} - 6\,g\,d_{0}\,d_{3} - 3\,d_{0}^{2}\,d_{3} - 6\,g\,d_{0}\,d_{3} - 3\,d_{0}\,d_{3}^{2} - 3\,d_{0}\,d_{0}^{2} -
                                                                                                                                                                              2^{2/3} \left(1+i\sqrt{3}\right) \left(2g^3-27bgn-3g^2n-3gm^2+2n^3-3g^2d_8+12gnd_8-3n^2d_8-3gd_8^2-3nd_8^2+2d_8^3-3g^2d_3-6gnd_3+6n^2d_3+12gd_8d_3-3g^2d_8-3gd_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^2+3g^2d_8^
```

Often one is not so much interested in the precise rate of invasion, but in whether a population can invade at all.

What is the invasion threshold?

Invasion Threshold

A solution of M-ALI=0

Invasion threshold A=0

Given by IMI=0

Invasion threshold

Example:
$$M = \begin{pmatrix} -d_{3} - d_{3} & 0 & b \\ g - d_{3} - m & 0 & 0 \end{pmatrix}$$

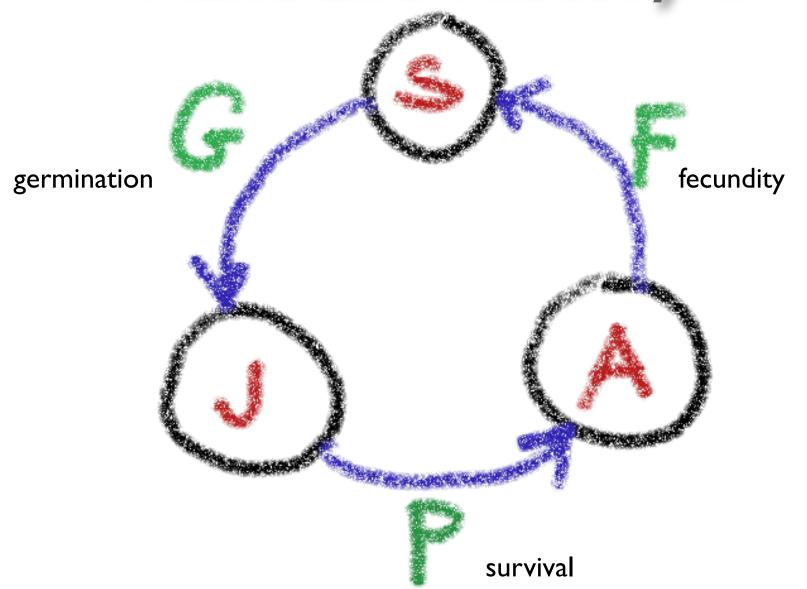
$$|M| = 0$$

$$-(d_{5}+g)(d_{3}+m) d_{1} + bgm = 0$$

$$\frac{b g m}{(a_{5}+g)(d_{3}+m) d_{1}} = \begin{pmatrix} R_{0} = 1 & reproduction ratio \\ R_{0} = 1 & reproduction ratio \end{pmatrix}$$

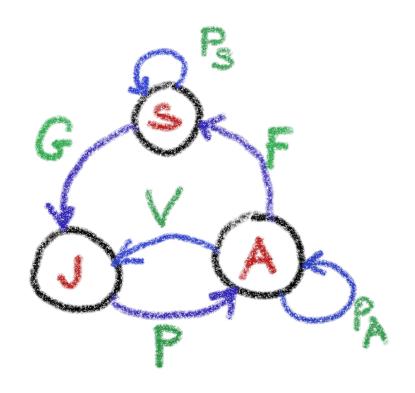
$$\frac{b g}{d_{5}+g} \frac{M}{d_{3}+m} - d_{1} = 0 \qquad per capita srowth rate$$

- Discrete time
- Three stages
 - Seeds S
 - Juveniles (non-reproducing) J
 - Adults (reproducing) A



$$\begin{cases} S_{e+1} \\ S_$$

M: Leslie matrix



- +Adult survival (perennial plants)
- +Seed survival (seed bank)
- +Vegetative reproduction

Analysis of linear models

Linear model

Solution
$$X_t = \sum_{i=1}^{n} c_i U_i \lambda_i^t$$
 λ_i^t λ_i^t i-th eigenvector λ_i^t i-th eigenvalue

Dominant eigenvalue λ

Solution converges to $\chi_{t} \propto u^{t}$

Population increases if $|\lambda| > 1$, decreases if $|\lambda| < 1$

Applications

Conservation biology

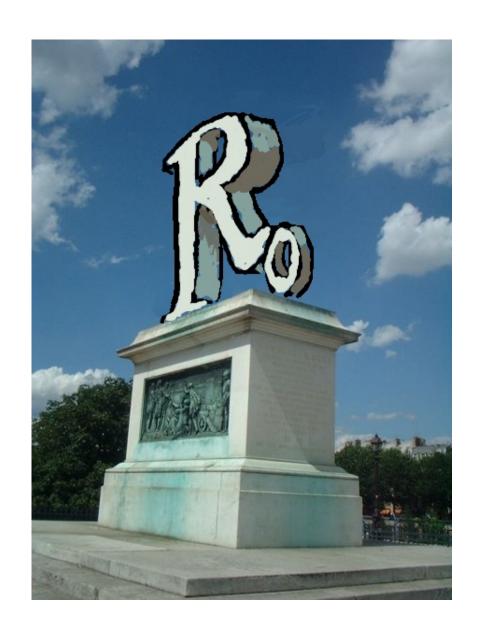
how can we prevent extinction of menaced populations?

Epidemiology

how can we prevent invasion of dangerous disease?

References

Caswell, H. (2001). Matrix Population Models. Construction, Analysis, and Interpretation. Sinauer, Sunderland, Mass, 2nd edition edition.



Measures of increase

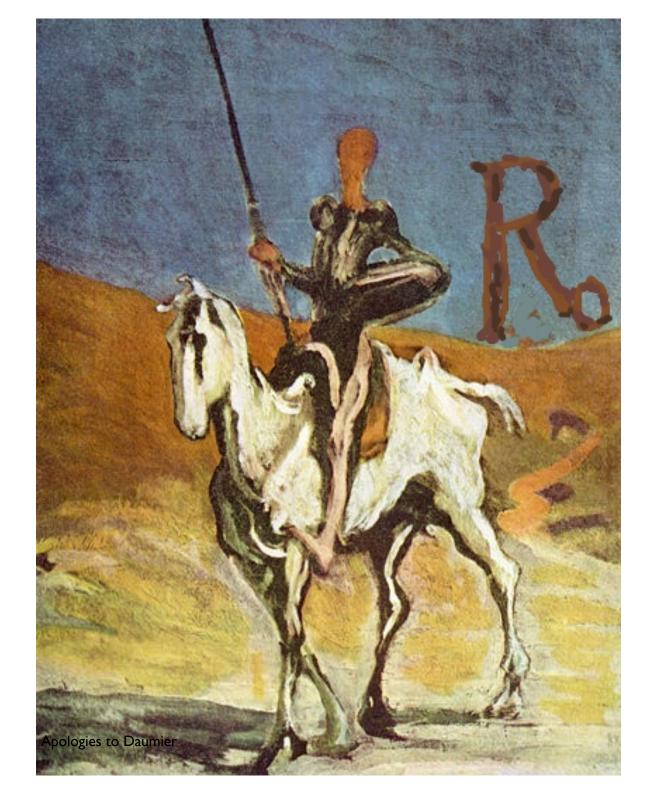
Subtle differences

- * λ rate of population increase
 - invasion continuous time : $\lambda > 0$
 - invasion discrete time : $\lambda > 1$
- $lacktriangleq R_0$ basic reproduction ratio
 - invasion : $R_0 > 1$

'typical' individual

- \bullet r net average rate of reproduction
 - invasion: r > 0

population property



Life History Theory

Generally

- environment is usually taken to be constant
- whereas in reality demographic rates are likely to be density dependent:

$$b = b(x,y,...), d = d(x,y,...)$$

Need to incorporate feedback

Life History Theory

Invasion in a dynamically changing environment

Realm of ...

Original Proposition

- Introduction into Adaptive Dynamics
- Application: Virulence Evolution
- Application: Kin Selection, Cooperation, and Units of Adaptation

Potential Topics

Synthetic Biology, Experimental Evolution
Mechanisms and Evolutionary Outcomes
Invasion Biology & Evolution
Genomics & Information Theory

Community Ecology (Ecosystem Dynamics)

Invasion

Evolution and Ecology

- Population Genetics
- Game Theory
- Life History Theory
- Community Ecology

Ecosystem Dynamics

Species are fixed entities

But there are potentially many of them

Which of these can coexist?

How does coexistence depend on their ecology?

How does it depend on external parameters?

Ecosystem Dynamics

Without ecological feedback

- only one species will dominate!
- species with the highest net rate of reproduction (r)

So how do we explain biodiversity?

Coexistence

Every species needs resources

- nutrients, light, space...
- species compete for these resources

Mathematical result:

- Number of species ≤ Number of resources
- if populations in ecological equilibrium (MacArthur in the 60s, Tilman 90s)

Coexistence

Nobody really knows how many different physical and chemical resources there are

But 100000000 different resources?

 100000000 is a low estimate of the number of currently existing species

Nonequilibrium Coexistence

Many if not most ecosystems are

- not in equilibrium
- but fluctuate

Fluctuating systems allow more species

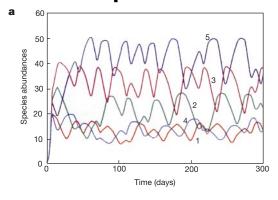
Armstrong & McGehee 1980s, Weissing & Huisman

Attractors

Every combination of species is represented by a dynamical system

Every dynamical system has its attractor(s)

equilibrium/periodic orbit/chaos



Hofbauer & Sigmund, Rinaldi

Permanence

In a permanent ecosystem no species will go extinct

Every participating species will invade when rare (ignoring 'Humpty Dumpty' effects)

Therefore to work out which species coexist we have to calculate their invasion exponent

Hofbauer & Sigmund, Rand

Invasion exponent

If a species' invasion exponent is positive it will invade the ecosystem

Invasion exponents can (in principle) be derived from the dynamical system

work out attractor without species



Invasion exponent

We can calculate invasion exponent λ of species i

- where x_i by considering the attractor of the n-1 species system $(x_i(t))$
- $r_i(t) = f(\dots, x_j(t), \dots) = f(E(t))$
- then $\lambda = \lim_{T \to \infty} \frac{1}{T} \int_0^T r_i(t) dt$

Ecosystem Dynamics

Caricature

- "Species dynamics depends on other species directly or indirectly
- Biodiversity is given by how many species from a given species pool can invade the community
- If no new species can invade, the community is saturated'

Jonathan (Joan) Roughgarden, Stuart Pimm

Ecosystem Dynamics

References

- Jonathan (now Joan) Roughgarden
 - Theory of Population Genetics and Evolutionary Ecology: An Introduction (1979)
- Josef Hofbauer & Karl Sigmund
 - The Theory of Evolution and Dynamical Systems (1988)

Invasion

Evolution and Ecology

- Population Genetics
- Game Theory
- Life History Theory
- Community Ecology